

Coordinate Systems

Lecture 3

Sections 2.2, 3.4

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Outline

- 1 Introduction
- 2 Transformations
- 3 The Coordinate Systems
 - Model Coordinates
 - World Coordinates
 - Eye Coordinates
 - Clip Coordinates
 - Normalized Device Coordinates
 - Viewport Coordinates
 - Window Coordinates
- 4 Assignment

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The Coordinate Systems

- The points we create are transformed through a series of coordinate systems before being drawn on the screen.
 - Model coordinates
 - World coordinates
 - Eye coordinates
 - Clip coordinates
 - Normalized device coordinates
 - Viewport coordinates
 - Window coordinates

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Changing Coordinate Systems

- In most cases, we change from one coordinate system to another by using a linear transformation.
- Each coordinate in the new system is a linear function of the coordinates in the old system.
- A linear transformation can be described by matrix multiplication:

$$X' = AX.$$

- A linear transformation always maps lines to lines, i.e., collinear points to collinear points.

Affine and Projective Transformations

Definition (Affine Transformation)

An **affine linear transformation** is a linear transformation that always maps

- Parallel lines to parallel lines.
- Intersecting lines to intersecting lines.

Definition (Projective Transformation)

A **projective linear transformation** is a linear transformation that may map

- Parallel lines to intersecting lines.
- Intersecting lines to parallel lines.

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The Coordinate Systems

- Most of the transformations between our coordinate systems are affine.

Model Coordinates

Definition (The Model Coordinate System)

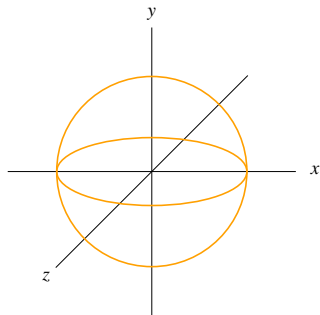
The **model coordinate system** is the coordinate system local to a specific object.

- An object is first drawn in its own model coordinate system.
- The origin and the axes are selected for convenience relative to that object.

Examples

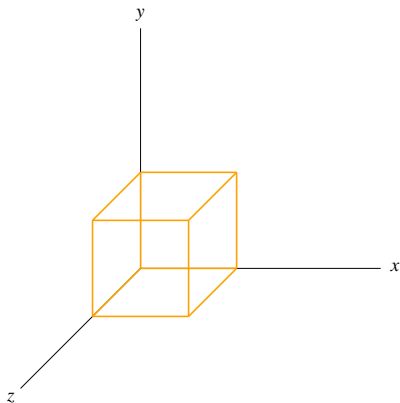
- Model coordinates for a sphere.
 - Place the origin at the center of the sphere.
- Model coordinates for a cube.
 - Place the origin at one corner.
 - Place the adjacent corners on the positive x -, y -, and z -axes.

Model Coordinates for a Sphere



Model Coordinate System for a Sphere

Model Coordinates for a Cube



Model Coordinate System for a Cube

Examples

- What would be a good model coordinate system for a cone?
- What would be a good model coordinate system for a cylinder?

Definition (The World Coordinate System)

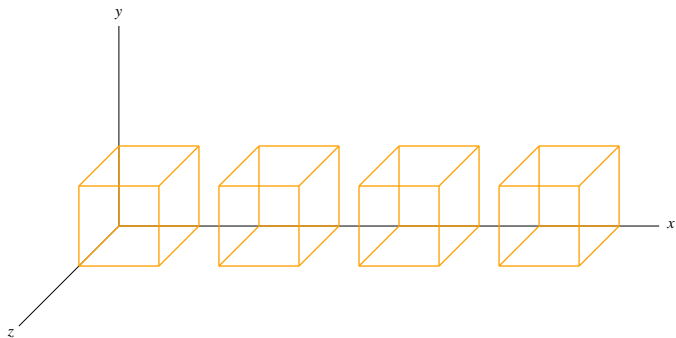
The **world coordinate system** is the system of coordinates used for building the 3-D scene as a single unified model.

- The origin and axes are chosen for convenience.

Example

- Imagine a scene with four cubes spaced out in a row.
- Each cube has its own model coordinate system.
- There is only one world coordinate system.

Five Cubes in a Row



World Coordinate System for a Collection of Cubes

World Coordinates

- Linear transformations are used to move the objects from their local coordinate systems to the world coordinate system.
- The most common ones are the following.
 - Translations
 - Rotations
 - Reflections
 - Scalings

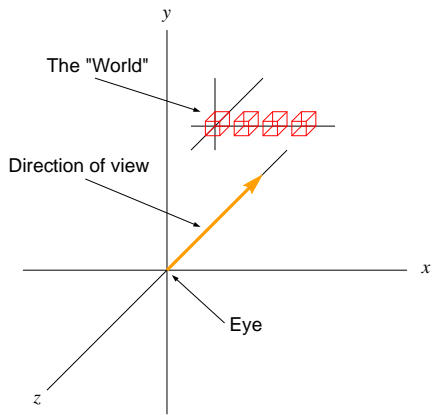
Eye Coordinates

Definition (The Eye Coordinate System)

In the **eye coordinate system**, the viewpoint (or “eye,” or “camera”) is located at the origin, looking down the negative z-axis.

- The eye coordinate determines the point of view and the direction of view.
- In world coordinates, the viewpoint is placed at the point from which we wish to view the scene.

The Eye Coordinate System



The Eye Coordinate System

Eye Coordinates

- Linear transformations are used to move the eye from the desired viewing position in world coordinates to the origin in eye coordinates.
 - Translations
 - Rotations

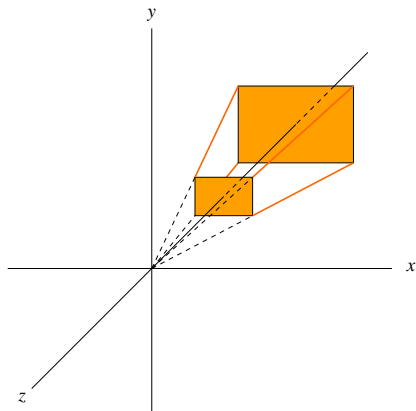
The View Frustum

Definition (The View Frustum)

The **view frustum** is a truncated rectangular pyramid with vertex at the eye and bases perpendicular to the line of sight.

- The view frustum is bounded by six planes.
 - The left and right planes.
 - The top and bottom planes.
 - The near and far planes.

The View Frustum

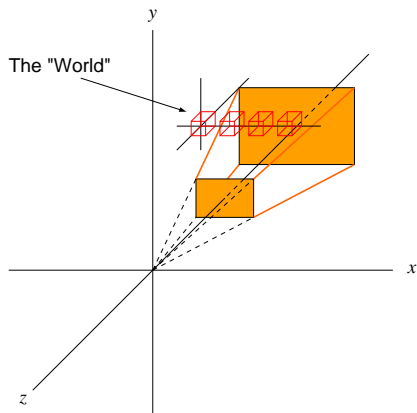


The View Frustum

The View Frustum

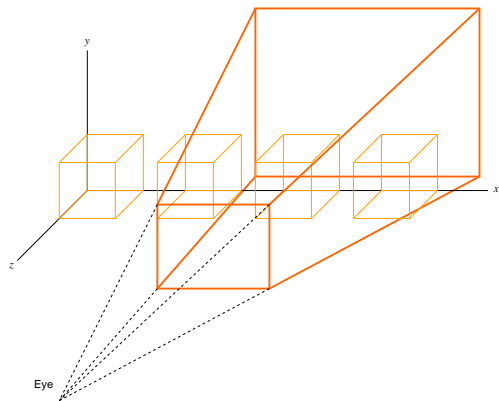
- The view frustum contains that part of the scene that will be visible on the computer screen.
- For technical reasons, the near and far planes eliminate objects that are either too near or too far away.

The View Frustum



The World in Eye Coordinates

The View Frustum

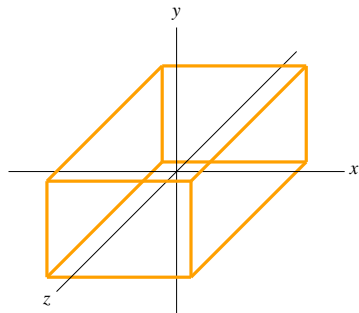


The View Frustum in World Coordinates

Clip Coordinates

- Eye coordinates are transformed to **clip coordinates** by another linear transformation.
- This transformation transforms the view frustum into a rectangular box.
- This transformation distorts angles and distances.
- In clip space, the GPU “clips” all objects, removing objects or parts of objects that are outside of the view frustum.

Clip Coordinates



The Clip Coordinate System

Normalized Device Coordinates

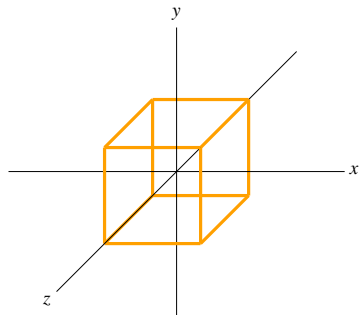
- **Perspective division** transforms clip coordinates into **normalized device coordinates**.
- The result is a cube that is centered at the origin.
- Each dimension ranges from -1 to 1 .

$$-1 \leq x \leq 1$$

$$-1 \leq y \leq 1$$

$$-1 \leq z \leq 1$$

Normalized Device Coordinates

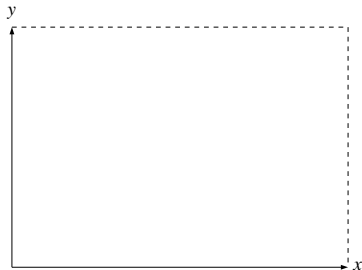


The Normalized Device Coordinate System

Viewport Coordinates

- The **viewport coordinate system** is two-dimensional, with the origin in the lower-left corner.
- The world scene in normalized device coordinates is “projected” onto the back face ($z = -1$) of the cube and then scaled.

Viewport Coordinates

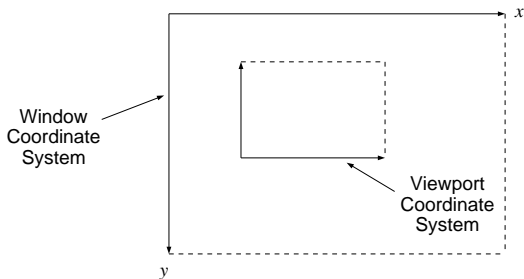


The Viewport Coordinate System

Window Coordinates

- The **window coordinate system** is like the screen coordinate system, except that
 - The origin is in the upper left corner.
 - Increasing y is down.

Window Coordinates



The Window Coordinate System

Window Coordinates

- Normally, the viewport will fill the window.
- Thus, the x -coordinates will be the same.

$$x_{\text{view}} = x_{\text{window}}.$$

- The y -coordinates will run in opposite directions.

$$y_{\text{view}} = \text{screen height} - y_{\text{window}}.$$

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Homework

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- Read Section 2.2 – a brief description of the coordinate systems.
- Read Section 3.4 – a more detailed description of the coordinate systems.